Is Piketty's "Second Law of Capitalism" Fundamental? by P. Krusell and A. Smith, Jr.

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Outline

Piketty's Theory

1. An accounting identity.
   The share of capital income in total income is:

$$\alpha = r \times \frac{K}{\tilde{Y}}$$

2. A long-run savings model (in net terms):

$$\frac{K}{\tilde{Y}} = \frac{\tilde{s}}{g}$$

3. An empirical observation:

$$r > g$$

4. A prediction
   With $r > g$ and net constant savings, capital share α rises

Krusell and Smith in this note

Compare Piketty's model with the textbook model

1. Textbook model
   - Constant gross saving rate s
   - Endogenous net savings rate $\tilde{s}(g)$ with $\frac{\partial \tilde{s}(g)}{\partial g} > 0$
   - $\tilde{s}(g)$ goes to 0 as g falls to zero
2. Piketty's model:
   - Constant net savings rate $\tilde{s}$
   - Endogenous gross savings rate $s(g)$ with $\frac{\partial s(g)}{\partial g} < 0$
   - $s(g)$ goes to 100% as g falls to zero
3. Optimization model with endogenous net and gross saving rates
   - Partial equilibrium with$g = 0$
   - General equilibrium with $g > 0$
4. U.S Data support the textbook model

Common elements

- Closed economy, constant population and technological growth at constant rate
- The accounting framework:

$$c_t + i_t = y_t$$

$$k_{t+1} = (1-\delta)k_t + i_i$$

$$y_t = F(k_t, z_t l)$$

- where F is CRS and the rate of growth of z_t is g

Textbook model: constant gross saving rate

- Assumptions
  - $F(k,\cdot)$ satisfies Inada conditions, in particular $F_1(k,\cdot) \to 0$ as $k \to \infty$
  - $i_t = sy_t$
- These assumptions deliver

$$k_{t+1} = (1-\delta)k_t + sF(k_t, z_t l)$$

- As $z_{t+1} = (1+g)z_t$, this becomes

$$(1+g)\hat{k}_{t+1} = (1-\delta)\hat{k}_t + sF(\hat{k}_t, l)$$

- Capital-to-gross output steady state ratio:

$$\frac{\hat{k}}{\hat{y}} = \frac{s}{g+\delta}$$

and along a balanced growth path $\frac{k_t}{y_t} = \frac{s}{g+\delta}$

Textbook model: implied capital-to-net ouput ratio

- Capital-to-gross output steady state (from previous slide):

$$\frac{\hat{k}}{\hat{y}} = \frac{s}{g+\delta}$$

- Capital-to-net output steady state (for later comparison with Piketty):

$$\frac{\hat{k}}{\hat{\tilde{y}}} = \frac{\hat{k}}{\hat{y}-\delta\hat{k}} = \frac{\frac{s}{g+\delta}}{1-\delta\frac{s}{g+\delta}} = \frac{s}{g+\delta(1-s)}$$

Textbook model: net saving rate is endogenous

- For later comparisons with Piketty's ratios, we also compute the net savings rate $\tilde{s}$ as:

$$\frac{\tilde{i}_t}{\tilde{y}_t} = \frac{sy_t - \delta k_t}{y_t - \delta k_t} = \frac{s - \delta \frac{k_t}{y_t}}{1 - \delta \frac{k_t}{y_t}}$$

- And on a balanced growth path

$$\tilde{s}(g) = \frac{s - \delta \frac{s}{g+\delta}}{1 - \delta \frac{s}{g+\delta}} = \frac{gs}{g + \delta(1-s)}$$

- Note that $\frac{\partial \tilde{s}(g)}{\partial g} > 0$ and $\tilde{s}(g)$ goes to 0 as g falls to zero (whereas Piketty assumes constant $\tilde{s}$)

Piketty's model: constant net savings rate

- Assumptions:
  - $\tilde{y} = \tilde{F}(\,) = F(k, \cdot) - \delta k$ satisfies $F_1(k, \cdot) - \delta \to 0$ as $k \to \infty$
  - Net saving rate is constant: $\tilde{i}_t = i_t - \delta k_t = \tilde{s}\,(y_t - \delta k_t)$
- These assumptions deliver

$$k_{t+1} = (1-\delta)k_t + i_t = k_t + \tilde{i}_t = k_t + \tilde{s}\,(y_t - \delta k_t) = k_t + \tilde{s}\tilde{F}(\,)$$

- As $z_{t+1} = (1+g)\,z_t$, this becomes

$$(1+g)\,\hat{k}_{t+1} = \hat{k}_t + \tilde{s}\tilde{F}(\hat{k}_t, l)$$

- Capital-to-net output steady state ratio (2nd law of capitalism):

$$\frac{\hat{k}}{\hat{\tilde{y}}} = \frac{\tilde{s}}{g}$$

and along a balanced growth path $\frac{k_t}{\tilde{y}_t} = \frac{\tilde{s}}{g}$

Piketty's model: implied capital-to-gross output

- From the law of motion

$$(1+g)\,\hat{k}_{t+1} = \hat{k}_t + \tilde{s}\tilde{F}(\hat{k}_t, l)$$

- his steady steady state gives:

$$g\hat{k} = \tilde{s}\left(F(\hat{k}_t, l) - \delta\hat{k}\right)$$

- and the ratio of Capita-to-gross output is:

$$\frac{\hat{k}}{\hat{y}} = \frac{\tilde{s}}{(g+\tilde{s}\delta)}$$

Piketty's model: gross saving rate is endogenous

- Consumption-output ratio is:

$$\frac{c_t}{y_t} = 1 - s = \frac{F(k_t, z_t l\ ) - i_t}{F(k_t, z_t l\ )} = \frac{\tilde{F}(k_t, z_t l\ ) - \tilde{i}_t}{F(k_t, z_t l\ )} = \frac{(1-\tilde{s})\,\tilde{F}()}{F(\,)}$$

- Then we can express the gross saving rate in terms of g:

$$s = 1 - \frac{(1-\tilde{s})\,\tilde{F}()}{F(\,)}$$

- From the ratios above $F(\,) = \frac{\hat{k}(g+\tilde{s}\delta)}{\tilde{s}}$ and $\tilde{F}() = \frac{\hat{k}g}{\tilde{s}}$:

$$s(g) = \frac{\tilde{s}\,(g+\delta)}{g+\tilde{s}\delta}$$

- where $\frac{\partial s(g)}{\partial g} < 0$ and $s(g)$ goes to 1 as g falls to zero (whereas Solow assumes constant s)

Steady state ratios for g=0, s=0.26, $\delta = 0.05$

	Solow	Piketty
K/Y^{gross}	$\frac{s}{g+\delta} \simeq 5$	$\frac{\tilde{s}}{(g+\tilde{s}\delta)} = 20$
K/Y^{net}	$\frac{s}{g+\delta(1-s)} = 7$	$\frac{\tilde{s}}{g} = \infty$

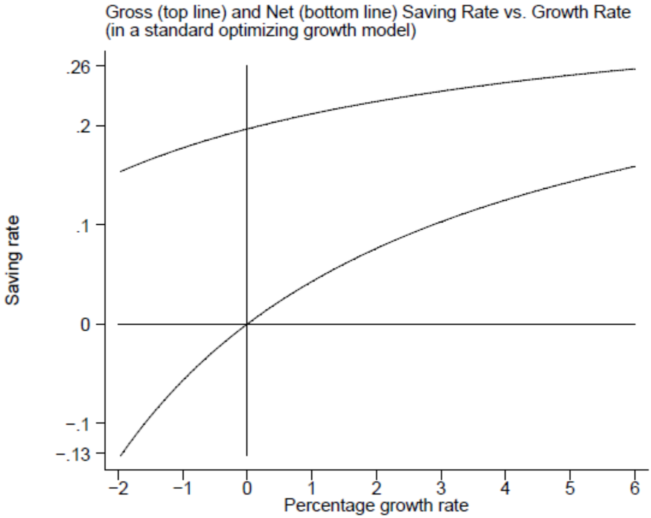
Saving rates

	Solow	Piketty
s^{gross}	*constant*	$\frac{\partial s(g)}{\partial g} < 0$
s^{net}	$\frac{\partial \tilde{s}(g)}{\partial g} > 0$	*constant*

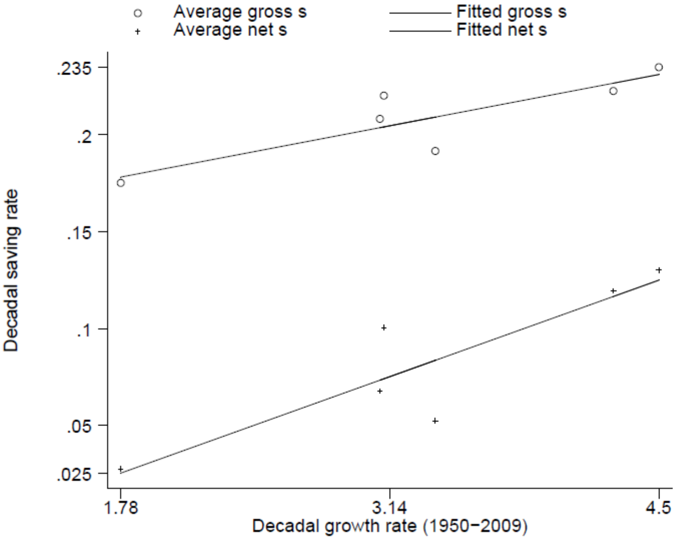
Partial equilibrium optimization model

- Partial equilibrium model with prices given, $g = 0$, and $\beta(1 + r - \delta) = 1$
  - Permanent-income behavior: asset holdings are constant and the consumer consumes the return on the asset plus the wage income
  - Net saving rate = 0 (like in Solow when g=0)

General equilibrium optimization model



U.S data



Average gross s
Average net s
Fitted gross s
Fitted net s
.235
.2
.15
.1
.05
.025
Decadal saving rate
1.78
3.14
4.5
Decadal growth rate (1950−2009)